Angles and Arc lengths - 3

Exercise - 3

Sol. - 1

1^c radian $= \frac{180}{\pi}$

$\frac{\pi}{6}$ rad. $= \frac{180}{\pi} \times \frac{\pi}{6} = 30^\circ$

$\frac{14\pi}{15}$ radian $= \frac{14\pi}{15} \times \frac{180}{\pi}$ (12)

$= 168^\circ$

$\frac{11}{18}\pi$ radian $= \frac{11}{18}\pi \times \frac{180}{\pi}$

$= 110^\circ$

$\frac{7\pi}{90}$ radian $= \frac{7\pi}{90} \times \frac{180}{\pi}$

$= 14^\circ$

Sol. - 2

(i) $1^\circ = 60'$

$1' = \left(\frac{1}{60}\right)^\circ$

$= \frac{1}{60} \times \frac{\pi}{180}$ radian

$= \frac{\pi}{10800}$ radian

(ii) $20^\circ = \frac{\pi}{180} \times 20$ radian

$= \frac{\pi}{9}$ radian

(iii) $135^\circ = \frac{\pi}{180} \times 135$ (9 3) radian

$= \frac{3\pi}{4}$ radian (×24)

Sol. 3

(i) $\boxed{\text{exterior angle} = \frac{360^\circ}{\text{no. of side}}}$

$$\text{exterior angle} = \frac{360^\circ}{40} = 9^\circ$$

$$1^\circ = \frac{\pi}{180} \text{ radian}$$

$$9^\circ = \frac{9 \times \pi}{180} = \frac{\pi}{20}$$

$$9^\circ = \frac{\pi}{20} \text{ radian}$$

interior angle of Polygon

$$= \pi - \frac{\pi}{20} = \frac{19}{20}\pi^\circ$$

(ii) $\boxed{\text{exterior angle} = \frac{360^\circ}{\text{no. of side}}}$

$$= \frac{360^\circ}{n} = \frac{2\pi}{n}$$

interior angle of n Polygon

$$= \pi - \frac{2\pi}{n}$$

$$= \frac{(n-2)\pi}{n} \text{ radian}$$

4) Perimeter of sector of circle $= 2r + s$

length of arc of semi-circle with radious $= \pi r$

ATQ $2r + s = \pi r$

$$2r + r\theta = \pi r$$

$$r(2 + \theta) = \pi r$$

$$\theta = \frac{22}{7} - 2$$

$$= \frac{22 - 14}{7} = \frac{8}{7} \text{ radian}$$

$$\theta = \frac{8}{7} \times \frac{180}{\pi}$$

$$= \frac{8}{7} \times \frac{180}{22} \times 7$$

$$\boxed{\theta = \frac{720}{11}} \text{ Ans}$$

$\frac{820}{11} = 65^\circ 27' 16''$

Sol. 5

Length of Pendulum = 8 m

angle θ = 1.5 radian

Length of arc = $r\theta$

$= (1.5 \times 8)$ m

$= 12$ m.

Sol. 6

length of minute hand of clock = r = 15 cm

angle in 60' = 360°

angle by minute hand in 40' = $\frac{360}{60} \times 40$

$= \frac{360}{60} \times 40$

$= 240^\circ$

$1^\circ = \frac{2\pi}{360}$ radian

$240 = \frac{\pi}{180} \times 240$

$\theta = \frac{4}{3}\pi$ radian

length of arc = $r \times \theta$

$= 15 \times \frac{4}{3}\pi$

$= 15 \times \frac{4}{3} \times \frac{22}{7}$

$= \frac{440}{7} = 62.85$ cm.

Sol. 7

Radious of circle = 50 cm.

length of arc = 10 cm

$\theta = \frac{l}{r} = \frac{10}{50}$ radian

$1^\circ = \frac{\pi}{180}$ radian

1 radian $= \frac{180}{\pi}$

$\frac{1}{5}$ radian $= \frac{1}{5} \times \frac{180}{\pi}^{36}$

$= \frac{36}{\pi} = \frac{36}{\frac{22}{7}}^{18} \times 7$

$= \frac{126}{11} = 11^\circ 27' 11''$

Sol. 8

$\theta = 31' = \left(\frac{31}{60}\right)^\circ$

1 radian $= \frac{180}{\pi}$

$\theta = \left(\frac{31}{60} \times \frac{\pi}{180}\right)$ radian

$= \frac{31\pi}{10800}$

$\theta = \frac{l}{r}$

$\frac{31}{10800} \times 360000 = l$

$l = \frac{31}{108} \times 360^{3400} \times \pi$

$= \frac{3100\pi}{3}$

$= \frac{3100}{3} \times \frac{22}{7}$

$= \frac{68200}{21}$ km.

diameter of moon $= \frac{68200}{21}$ km.

Sol. 9

Radius of curve = 750 m

distance travelled by train in 1 hour = 30 km

$= 30 \times 1000$ m

In 1 sec., the distance cover by train $= \frac{30 \times 1000}{60 \times 60}$ m

$= \frac{25}{3}$ m

distance cover in 10 seconds

$= 10 \times \frac{25}{3} = \frac{250}{3}$ m

$\therefore$ The required angle $\theta = \frac{l}{r}$

$= \frac{250}{3} \times \frac{1}{750}$

$= \frac{1}{9}$ radian

Sol. 10

$\angle POQ = 70°$ $QP = 910$ cm

$\angle POQ = \frac{\text{arc } PQ}{\text{radius}}$

$70° = \frac{\text{arc } PQ}{910}$

arc $PQ = 70 \times 910$

$= 56700$ radian

$= \frac{\pi}{180} \times 56700$

$= \frac{22}{7 \times 180} \times 56700$

$= 990$ cm.

Sol → 11

ar of sector $= \frac{1}{2} r^2\theta$

$6.024 = \frac{1}{2} \times r^2 \times 36 \times \frac{\pi}{180}$

$\frac{180 \times 6.024 \times 2}{36 \times \cancel{22}} = r^2$

$\frac{180 \times 6.024 \times 7 \times 2}{36 \times 22} = r^2$

$r = \sqrt{5 \times 6.024 \times 7}$

$r = 3.09$ cm.

Sol → 12 Radius of sector of circle $= 5$ m

length of arc $= 8$ m.

angle $\theta = ?$

$\theta = \frac{l}{r} = \frac{8}{5}$ Radian

area of sector $= \frac{1}{2} r^2\theta$

$= \frac{1}{2} \times 5 \times 5 \times \frac{8}{5}$

$= 20\ m^2$

Sol - 13

$\theta = \frac{8\pi}{9}$ $r = 45$ cm.

length of arc $= l = r\theta$

$= 45 \times \frac{8\pi}{9}$

$= 5 \times 8\pi = 40\pi$ cm

(ii) area of windscreen $= 55 \times 100\ cm^2$

area of windscreen not cleaned

$= 55 \times 100 - \frac{1}{2}\theta r^2\ cm^2$

$= 55 \times 100 - \frac{1}{2} \times 45 \times 45 \times \frac{8}{9} \times \frac{22}{7}$

$= 5500 - \frac{19800}{7}$

$= 5500 - 2828.57$

$= 2671.43\ cm^2$

Sol → 14

area of sector $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 24 \times 24 \times 60^\circ$

$= 12 \times 24 \times 60 \times \frac{\pi}{180}$

$= 4 \times 24 \times \pi$

$= 96 \times \frac{22}{7}\ cm^2$

area of shaded region = ar. of sector − ar. of $\triangle$

$= 96 \times \frac{22}{7} - \frac{1}{2} \times OA \times OB \times \sin 60$

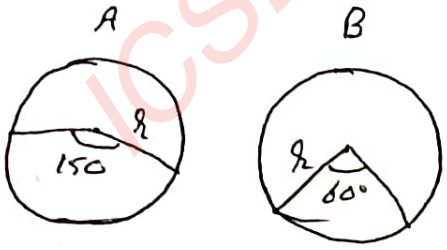
$= 96 \times \frac{22}{7} - \frac{1}{2} \times 24 \times 24 \times \frac{\sqrt{3}}{2}$

$= 96 \times \frac{22}{7} - 6 \times 24 \times 1.732$

$= 301.71 - 249.41$

$= 52.3\ cm^2$

Sol. → 15



area of major sector in dia. A

$= \frac{1}{2} r^2 \theta = \frac{1}{2} \times r^2 \times 150 \times \frac{\pi}{180}$

$= \frac{1}{2} r^2 \times \frac{5}{6} \pi$

area of minor sector in dia. B

$= \frac{1}{2} r^2 \times \frac{\pi}{3}$

Required ratio $= \frac{\text{area of major area}}{\text{area of minor area}}$

$= \frac{\frac{1}{2} \times r^2 \times \frac{5}{6} \pi}{\frac{1}{2} \times r^2 \pi / 3}$

$= \frac{5}{6} \times \frac{3}{1} = 5 : 2$ ← Ans

— The END —